MATH 119 Chapter 9

0.1 Definition 1

i)The set of all pairs of real numbers (x, y) is denoted by $\mathbb{R}^2$.

ii) The set $\mathcal{B}_r(a, b)$ of pairs $(x, y) \in \mathbb{R}^2$ inside the circle of radius r with center in (a, b) is called a disk. It means $\mathcal{B}_r(a, b)$ consists of all pairs $(x, y) \in \mathbb{R}^2$ satisfying $x^2 + y^2 < r$.

0.2 Definition 2: Relative Maximum and Relative Minimum

Let $\mathcal{D} \subseteq \mathbb{R}^2$ be the domain of the function f.

i) A function of two variables has a **relative minimum at** (a, b) if $f(x, y) \geq f(a, b)$ for all points (x, y) in some $\mathcal{B}_r(a, b)$ contained in $\mathcal{D}$. The number $f(a, b)$ is called a **relative minimum value of** f.

ii) A function of two variables has a **relative maximum at** (a, b) if $f(x, y) \leq f(a, b)$ for all points (x, y) in some $\mathcal{B}_r(a, b)$ contained in $\mathcal{D}$. The number $f(a, b)$ is called a **relative maximum value of** f.

0.3 Definition 3: Critical Points

Consider a function f whose first order partial derivatives, f_x and f_y exist at a point (a,b) in $\mathbb{R}^2$. If

$$f_x(a, b) = 0 \qquad \text{and} \qquad f_y(a, b) = 0,$$

then the pair (a, b) is called a **critical point of** f**.**

0.4 Theorem 1: Necessary Condition for a Relative Extrema

If the following two conditions are verified:
i) f has a local maximum or minimum at (a, b)
. and
ii)The first order partial derivatives $f_x(a, b)$ and $f_y(a, b)$ exist, then

$$f_x(a, b) = 0 \quad \text{and} \quad f_y(a, b) = 0$$

0.5 Theorem 2: Test for Relative Extrema

Consider a function f defined on a domain $\mathcal{D}$ of $\mathbb{R}^2$, then if

i) There is a point (a, b) in $\mathcal{D}$, such that $f_x(a, b) = 0$ and $f_y(a, b) = 0$. It means the point (a, b) is a critical point of f.

ii) The second-order partial derivatives f_{xx}, f_{xy}, and f_{yy} are continuous in a disk $\mathcal{B}_r(a, b)$ contained in $\mathcal{D}$.

iii) $D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - (f_{xy}(a, b))^2$,

then

a) If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f reaches a relative minimum $f(a, b)$ at the critical point (a, b).

b) If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f reaches a relative maximum $f(a, b)$ at the critical point (a, b).

c) If $D(a, b) < 0$, then $f(a, b)$ is not a relative extrema and there is a **saddle point on the graph of** f at the critical point (a, b).

d) If $D(a, b) = 0$ the test gives no information.